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Premia in Forward Foreign Exchange as Unobserved Components: A Note

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We reconsider the signal-extraction approach to measuring premia in the pricing of forward foreign exchange, put forward by Wolff, in which the difference between the forward rate and the associated future spot rate is modeled as an autoregressive moving average (ARMA) model for the risk premium buried in a white-noise forecast error. We point out that an ARMA model for the risk premium is not always identifiable from information on the difference between the forward rate and the future spot rate only. We present solutions to the problem of identification and show how the model for the risk premium can be estimated in a direct way, provided that the identification problem is solved. For reason of comparison, we use the series analyzed by Wolff to estimate the models for risk premia. The results confirm the earlier finding that premia in forward exchange exhibit a certain degree of persistence over time.

KEY WORDS: ARMA models; Exchange risk; Identification; Kalman filter.

The pricing of forward foreign exchange has received considerable attention in the finance literature in recent years. Time-varying risk premia in the pricing of forward contracts have been documented and characterized in several studies—for example, by Baillie and Bollerslev (1990), Domowitz and Hakkio (1985), Fama (1984), Hansen and Hodrick (1980, 1983), Hodrick and Srivastava (1986), and Wolff (1987).

Along with other authors, Wolff (1987) divided the forward foreign exchange rate observed at time t for currency to be delivered at $t + 1$ into an expected future-spot-rate component and a premium component

$$f_t^{t+1} = E_t s_{t+1} + P_t, \tag{1}$$

where f_t^{t+1} is the natural logarithm of the forward rate at time t for an exchange at $t + 1$, $E_t s_{t+1}$ denotes the expectation of the log of the spot rate at $t + 1$, $E[s_{t+1}|I_t]$, conditional on all information I_t available at t, and P_t is a premium term. Subtracting s_{t+1} from both sides of Equation (1) and defining $v_{t+1} \equiv E_t s_{t+1} - s_{t+1}$, we obtain

$$f_t^{t+1} - s_{t+1} = P_t + v_{t+1}, \tag{2}$$

where v_t is an uncorrelated, zero-mean sequence with variance σ_v^2. As noted by Wolff (1987), Equation (2) states that the forecast error resulting from the forward rate as a prediction of the future spot rate consists of a premium component and a white-noise error term due to the arrival of new information between t and $t + 1$ concerning the spot rate at $t + 1$. The premium component P_t is referred to as the signal that will be assumed to be generated by an autoregressive moving average (ARMA) or autoregressive integrated moving average (ARIMA) model. The component v_{t+1} is a noise added to the signal, which has to be extracted. The advantage of the signal-extraction approach proposed by Wolff (1987) is that the expectation of s_{t+1}, given information up to period t, does not have to be modeled. One can, for instance, abstract from announcement effects, which affect both f_t^{t+1} and $E_t s_{t+1}$ without affecting P_t.

Assuming that P_t is generated by an ARIMA model is consistent with structural models that have been studied in the literature. For instance, under the assumptions made by Domowitz and Hakkio (1985) that the utility functions are of the Cobb–Douglas form and that the log of the initial endowments with money and goods in both countries follow independent AR(1) processes, Lucas's (1982) international model leads to the following expression for the risk premium:

$$f_t^{t+1} - E_t s_{t+1} = -.5[\mathrm{var}_t(m_{t+1}) - \mathrm{var}_t(n_{t+1})], \tag{3}$$

where $\mathrm{var}_t(m_t)$ and $\mathrm{var}_t(n_t)$ denote the conditional variances of the logarithms of nominal domestic and foreign money balances, respectively. It is straightforward to show that if, for instance, these conditional variances are generated by a first-order generalized autoregressive conditional heteroscedastic (GARCH) process, the premium in (3) is generated by an ARMA(2,1) model. If both conditional variances follow an ARCH(2) process, the premium will be generated by an MA(1) model.

In a similar way, ARMA processes for the risk premium can also be derived from Hodrick's (1989) model.

The empirical findings for the structural models of the premium are unsatisfactory (e.g., see Baillie and Bollerslev 1990; Domowitz and Hakkio 1985; Giovannini and Jorion 1989; Hodrick 1989). Possible reasons are the oversimplification of the model, the omission of relevant explanatory variables from the variance function (see Hodrick 1989), the misspecification of the functional form of the conditional variance (see Pagan and Ullah 1988), and the choice of the sampling frequency. Note that the analysis of data at a frequency that does not coincide with the frequency in the data-generating process can distort the empirical results (compare Drost and Nijman 1990). Such problems do not arise if the premium is modeled as an ARMA process because the class of ARMA processes is closed under temporal aggregation (e.g., see Palm and Nijman 1984). Given that structural models do not yield satisfactory empirical results, it is appropriate to adopt an ARMA model that allows one to analyze important features of the premium such as time variation and persistence under weaker assumptions than those underlying structural models.

This article goes beyond the methodology proposed by Wolff (1987). First, it investigates the issue of identification of the parameters of the model for the risk premium from the difference between the forward rate and the associated future spot rate only. Second, the time series used by Wolff are analyzed using standard time series methods rather than the Kalman filter, which makes it much more straightforward to handle the identification problem. Third, it is briefly discussed how the risk premium can be estimated in a direct way provided that the identification problem is solved. The note is organized as follows. Section 1 is devoted to the specification, identification, and estimation problem of models in which the premium is an unobserved component generated by an ARMA model. Section 2 contains the results of an empirical study of premia in this model for the three bilateral exchange rates analyzed by Wolff (1987). Section 3 concludes this note.

1. MODELING THE PREMIUM AS AN UNOBSERVED COMPONENT

To specify the model for $f_t^{t+1} - s_{t+1} = y_t$ completely, let us assume that (1) and (2) hold and that P_t is generated by the following ARMA(1, 1) model:

$$(1 - \phi L)P_t = (1 - \theta L)a_t, \quad (4)$$

where L is the lag operator, $|\phi| < 1$, $|\theta| \leq 1$, $\phi \neq \theta$, and a_t is a white noise with mean 0 and constant variance σ_a^2 and is uncorrelated with $v_{t'}$, for all t and t'. We assume that $Ea_t v_{t+i} = 0$ ($\forall$ i). This assumption could be relaxed in favor of allowing for correlation between past forecast errors and the current premia. Substitution of expression (4) into (2) yields the following model for y_t:

$$(1 - \phi L)y_t = (1 - \theta L)a_{t-1} + (1 - \phi L)v_t, \quad (5)$$

which is an ARMA(1, 1) model because the right side of Model (5) can be expressed as an MA(1) process with innovation ε_t yielding

$$(1 - \phi L)y_t = (1 - \omega L)\varepsilon_t, \quad (6)$$

where $|\omega| \leq 1$ and ε_t has mean 0 and variance σ_ε^2. A comparison of (5) and (6) immediately shows that the well-known order condition for identification is not satisfied because the *reduced form* in (6) contains three parameters that are identified under the usual regularity conditions but the *structural form* in (5) contains four parameters.

The second moments of the right side of (5) and (6) are identical. The variance and the first-order autocovariance are given by, respectively,

$$(1 + \theta^2)\sigma_a^2 + (1 + \phi^2)\sigma_v^2 = (1 + \omega^2)\sigma_\varepsilon^2 \quad (7a)$$

and

$$-\theta\sigma_a^2 - \phi\sigma_v^2 = -\omega\sigma_\varepsilon^2. \quad (7b)$$

The higher order autocovariances are 0. With ϕ being derived from the AR part of (6), the three parameters θ, σ_a^2, and σ_v^2 cannot be uniquely determined from two relations in (7), and therefore they are not identifiable. Similarly, when $\phi = 0$, the expressions in (7) specialize accordingly. The three unknown parameters θ, σ_a^2, and σ_v^2 cannot be uniquely determined from the information in Ey_t^2 and $Ey_t y_{t-1}$ (or equivalently from σ_ε^2 and ω). This implies that the log-likelihood function does not have a well-defined maximum. The values for the maximum of the likelihood function reported by Wolff (1987) for the dollar/mark and the dollar/yen rates therefore have to be an artifact of numerical approximations in finite samples or imprecision caused by the complexity of the Kalman-filter algorithm.

Although σ_a^2 and σ_v^2 are not identified, it is possible to obtain upper and lower bounds for these variances. From (7a) and (7b) and $\phi = 0$, one obtains

$$\sigma_a^2 = \theta^{-1}\omega\sigma_\varepsilon^2 \quad (8a)$$

and

$$\sigma_v^2 = \sigma_\varepsilon^2(1 + \omega^2) - \omega\sigma_\varepsilon^2(\theta^{-1} + \theta). \quad (8b)$$

If $\omega < 0$, for example, θ will be negative as well, and as $|\theta| \leq 1$, Equations (8a) and (8b) yield estimable bounds on σ_a^2 and σ_v^2,

$$|\omega|\sigma_\varepsilon^2 \leq \sigma_a^2 \leq \sigma_\varepsilon^2 \quad (9a)$$

and

$$0 \leq \sigma_v^2 \leq \sigma_\varepsilon^2(1 + \omega)^2. \quad (9b)$$

Similar bounds can be obtained if $\omega \geq 0$. Along similar lines, it is possible to derive upper and lower bounds for the variances σ_v^2 and σ_a^2 in the ARMA(1, 1) model. It is easy to show that if $\phi \geq \omega \geq 0$ or if $\omega \leq \phi \leq 0$

the variances σ_v^2 and σ_a^2 satisfy the following bounds:

$$0 \leq \sigma_v^2 \leq \sigma_\varepsilon^2(1 + \omega)^2/(1 + \phi)^2 \quad (10a)$$

and

$$\sigma_\varepsilon^2[(1 + \omega^2)\phi - (1 + \phi^2)\omega] \div (1 + \phi)^2 \leq \sigma_a^2 \leq \sigma_\varepsilon^2. \quad (10b)$$

Note that if we let ϕ go to 0, we obtain again the MA(1) model for y_t with lower bound $|\omega|\sigma_\varepsilon^2$ for σ_a^2 and upper bound $(1 + \omega^2)\sigma_\varepsilon^2$ on σ_v^2. For simplicity reasons, we have just considered the case in which the premium is generated by an ARMA(1, 1) model. In the general case in which P_t is generated by an ARMA(p, q) process, it can be shown that y_t follows an ARMA(p, max(p, q)) process, the parameters of which are identified if $p > q + 1$ (e.g., see Hotta 1989; Maravall 1979).

2. EMPIRICAL RESULTS FOR THE PREMIUM AS UNOBSERVED COMPONENT

In this section we study the premia for the three bilateral exchange rates. For reasons of comparison, we use the series analyzed by Wolff (1987)—the dollar/pound, dollar/mark, and dollar/yen exchange rates. Thirty-days forward rates and subsequently observed spot rates are taken from the Harris Bank Data Base supported by the Center for Studies in International Finance at the University of Chicago. The rates are Friday closes, sampled at four-week intervals covering the period April 6, 1973, to July 13, 1984.

As discussed by Wolff (1987), the autocorrelation and partial autocorrelation functions for $y_t = f_t^{t+1} - s_{t+1}$ suggest an ARMA(1, 1) model in the dollar/pound case and MA(1) models in the dollar/mark and dollar/yen cases. These models are also chosen when Akaike's information criterion is used. Estimates of these models are presented in Table 1. Large-sample standard errors are given in parentheses. In all cases a very insignificant constant has been removed prior to estimation. As shown in Section 1, the ARMA(1, 1) model for y_t in the dollar/pound case is consistent with either an AR(1) or an ARMA(1, 1) model for the premium. If one is willing to restrict θ a priori to be equal to 0—that is, to choose the AR(1) model—both σ_v^2 and σ_a^2 can be identified from ϕ, ω, and σ_ε^2. The implied estimates of σ_a^2, the variance of the innovation in the premium, and σ_v^2, the variance of the innovation in the exchange rate, are $2.77 * 10^{-4}$ and $3.06 * 10^{-4}$, respectively, which are of course very close to the estimates reported by Wolff (1987), who estimated the same model along different lines. If the premium is not restricted to being generated by an AR(1) model but is allowed to be generated by an ARMA(1, 1) model, Equations (10a) and (10b) yield bounds on the two variances for the dollar/pound case: $.60 * 10^{-4} \leq \sigma_a^2 \leq 6.15 * 10^{-4}$ and $0 < \sigma_v^2 \leq 4.36 * 10^{-4}$.

The MA(1) models for y_t in the dollar/mark and dollar/yen cases are only consistent with MA(1) models for the premium. In these cases, the parameters θ, σ_a^2, and σ_v^2 are not identified, but the bounds in (8) are available. Note that, as in the work of Wolff (1987), in all cases there is evidence of nonnegligible time variation of the premium, although his claim that the two variances roughly coincide has to be dropped.

Alternatively, identification of the parameter of a moving average process for P_t can be achieved by modeling the expectation of the spot rate conditional on past information. For instance, assuming that the log of the spot rate follows a random walk,

$$s_{t+1} = s_t + v_{t+1};$$

$$\text{that is, } E[s_{t+1}|s_t, s_{t-1}, \ldots] = s_t, \quad (11)$$

the risk premium in (1) becomes observable and equal to the forward premium $f_t^{t+1} - s_t = P_t$, provided that

$$E[s_{t+1}|I_t] = E[s_{t+1}|s_t, s_{t-1}, \ldots]. \quad (12)$$

Choosing the random-walk specification (11) to extract the signal P_t can be interpreted as choosing a canonical representation for the signal, which maximizes the variance of the noise term v_{t+1} in (1). The condition (12) implies that semistrong-form market efficiency coincides with weak-form market efficiency. In the Domowitz–Hakkio (1985) model, the expected future spot rate satisfies the relationship $E[s_{t+1}|I_t] = s_t - (1 - \rho_m)m_t + (1 - \rho_n)n_t$, with ρ_m and ρ_n being the coefficients of the first-order autoregressive process for domestic and foreign money balances m_t and n_t, respectively. Therefore, Assumption (12) holds true when the processes for m_t and n_t have a unit root. There is a considerable body of evidence that indicates that spot exchange rates follow patterns that are close to random walks (e.g., see Meese and Rogoff 1983). The random-walk assumption, therefore, appears to be a good first ap-

Table 1. Least Squares Estimation of ARMA Model (9) for $f_t^{t+1} - s_{t+1}$ and Implied Bounds on Parameters (σ_ε^2 is an upper bound on σ_a^2)

	ϕ	ω	σ_ε^2 ($\times 10^4$)	Lower bound σ_a^2 ($\times 10^4$)	Upper bound σ_v^2 ($\times 10^4$)
Dollar/pound	.458 (.248)	.228 (.265)	6.15	.60	4.36
Dollar/mark	0	−.167 (.086)	9.48	1.58	6.58
Dollar/yen	0	−.195 (.082)	8.28	1.62	5.37

proximation. Assumption (12) is much more questionable, however.

For the dollar/mark and the dollar/yen spot rates, a random-walk model appears to be appropriate because none of the autocorrelations and partial autocorrelations are significantly different from 0 at the 5% level. For the dollar/pound spot rate, an ARI(1, 1) model could be entertained as well, because the first serial correlation coefficient is significantly different from 0. Moreover, applying an augmented Dickey–Fuller test to s_t, the hypothesis of a unit root cannot be rejected yielding additional evidence for the appropriateness of the random-walk model for these series.

As mentioned previously, under the assumptions that the marginal process for s_t is a random walk and that Assumption (12) holds, the premium $P_t = x_t = f_t^{t+1} - s_t$ is observable. The autocorrelations and partial autocorrelations for the dollar/pound forward premium x_t are consistent with an AR(1) model selected by Wolff (1987). This finding is further evidence in favor of the outcome from the analysis of the risk premium as an unobserved component, although the estimate $\hat{\phi} = .855$ is larger than the estimate obtained previously. For the dollar/mark and the dollar/yen rates, the autocorrelations of x_t slowly die out. ARMA(1,1) models for x_t appear to be more appropriate than an AR(1) or an MA(1) model. Their parameter estimates have values that when substituted for P_t in (1) could lead to cancelling of common roots, explaining why an MA(1) process was found to be appropriate for y_t. Details of the empirical analysis can be made available on request.

3. CONCLUDING REMARKS

In this note we reconsidered a methodology put forward by Wolff (1987) in which the forward-risk premia are modeled as unobserved components generated by an ARMA(p, q) process and buried in white noise. ARMA processes for the risk premium are consistent with several structural models presented in the literature, but their use requires fewer strong assumptions than the structural models generally do. Moreover, provided that the ARMA model for the premium can be estimated, the premium can be extracted from the series $f_t^{t+1} - s_{t+1}$ using the Kalman filter or the Wiener–Kolmogorov filter. When the parameters of the process of the premia cannot be identified from observations on $f_t^{t+1} - s_{t+1}$ only, inequality restrictions for some unidentified parameters can be derived or additional information has to be brought in, either in the form of restrictions derived from an underlying structural model or from information in other time series.

For the dollar/pound rate, our empirical analysis confirms the choice of an AR(1) process for P_t made by Wolff (1987). For the dollar/mark and the dollar/yen rate, the results for the series $f_t^{t+1} - s_t$ suggest an ARMA(1, 1) model for P_t. The parameters of this model can be identified by making the assumption that $E_t s_{t+1} = s_t$, which allows us to measure the risk premium by $f_t^{t+1} - s_t$. Our findings confirm Wolff's earlier result that premia in the pricing of forward rates show a certain degree of persistence over time. His conclusion that the innovation variance of the premium σ_a^2 and the variance of the innovation in the spot rate are of the same order of magnitude is not confirmed, however.

Compared with the Kalman-filter technique adopted by Wolff (1987), the use of the ARMA representation of the unobserved component model has the following advantages: (a) It yields directly testable restrictions on the second moments of the observed series. (b) The conditions for identification can be easily verified. (c) Provided that the parameters are identified, they can be estimated fairly easily using a minimum distance estimator, which first estimates the unrestricted ARMA model for y_t and then determines estimates of the parameters of the processes for P_t and σ_v^2 by minimizing a quadratic distance between unrestricted estimates of the parameters of the process for y_t and the restricted parameters (e.g., see Kodde, Palm, and Pfann 1990). (d) For stationary and nonstationary processes, an estimate for the signal P_t can be obtained using the Wiener–Kolmogorov filter (e.g., see Maravall 1988) in the context of other financial markets. Identification and estimation of term premia in the term structure of interest rates, for instance, can be structured within our framework.

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